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Generalized mean-field theory relating helix tilt in a bilayer to lipid disorder

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Abstract

We present a generalized theory relating the helix tilt angle in a bilayer to lipid disorder. In doing so, we consider a theory performed earlier [Biophys. Chem., 86 (2000) 79] and generalize it within a nonextensive formalism. The generalized theory provides a method to compare the rotational barriers for different helices in lipid bilayers, accounting for long-range interactions via a parameter which is called "entropic index". The results obtained could lead to point out future experiments which might shed light on lipid–protein interactions.

Keywords: Tsallis thermostatistics; Membrane protein; Helix tilt; Lipid disorder; Mean-field theory

1. Introduction

Transmembrane helical proteins and their phospholipid bilayer have been investigated theoretically [1–4], computationally [5–10] and experimentally [11–23]. Also, some experiments have shown that many properties are influenced by the lipid–protein interactions, for example, the helix tilt angle [11,12], the activity of lactose permease [11], the folding rate of bacteriorhodopsin [14], lipid disorder [15–17] and dynamics [18], the bilayer thickness [19]. However, theoretical studies have imposed on either the phospholipid bilayer [1] or the general phase behaviour of the lipid–protein system [2,3], and less emphasis has been placed on the protein [4]. Therefore, a mean-field theory on helices in a bilayer has been presented in Ref. [24] where the angle between the helical axis and the bilayer normal has been noticed. In this manner, a comparison has been made between experimental and theoretical results for lactose permease in Ref. [24] and it has been noted that the agreement has deteriorated towards higher values of the average disorder parameter of the lipid bilayer, which could be due to interactions neglected in the Hamiltonian. Therefore, in this study, we would like to present a new generalized theory which is useful to study the systems having long-range interactions. In this sense, it could be possible to improve the standard theory.

We organise the paper as follows: First, we summarize the standard mean-field theory and a nonextensive formalism which is called Tsallis formalism. Then we generalize the standard theory and finally, the variation of the tilt angle between the helical axis and the bilayer normal with the average disorder parameter of the lipid bilayer is plotted for some values of the entropic index, coming from the definition of Tsallis statistics. The system considered in Ref. [24] is illustrated in Fig. (1). A helix is represented as a cylinder. θ is the tilt angle between the helical axis and the bilayer normal. The system is cylindrically symmetric. So the thermal average of the helix tilt angle is given by

$$\langle\theta\rangle = \frac{\int_0^{\pi} \sin\theta\theta\exp(-H/kT)\mathrm{d}\theta}{\int_0^{\pi} \sin\theta\exp(-H/kT)\mathrm{d}\theta,} \tag{1}$$

where H is Hamiltonian of the system, k is the Boltzmann constant and T is the absolute temperature. In analogy with a polymer in an orientation field [25], it has been proposed for the Hamiltonian of the system:

$$H = -\frac{\alpha\langle S_{\mathrm{lipid}}\rangle\cos\theta}{kT} = -\gamma\langle S_{\mathrm{lipid}}\rangle\cos\theta, \tag{2}$$

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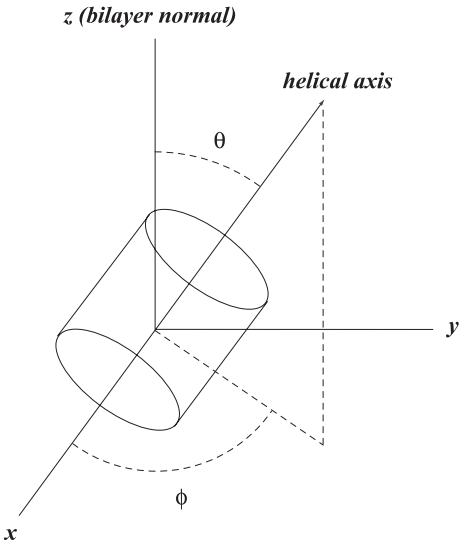

Fig. 1. The structure of the helix, which is represented by a cylinder. The lipid bilayer spreads along the x–y plane.

where $\alpha = \gamma/kT$ and γ is a positive constant whose magnitude depends on the protein. γ can be determined by fitting to the experimental results, and is related qualitatively to how much energy is needed to rotate the helix in a bilayer. $\langle S_{\text{lipid}} \rangle$ is the average disorder parameter of the lipid bilayer [26] that can be evaluated from NMR [26] or IR [11] spectroscopy, independently of γ.

The motivation of Eq. (2) was described as follows: the cosine function ensured that $\theta = 0$, so that it modelled the energetic cost from the disturbing of lipid packing [6] and discouraged the helix from flipping across membrane. $\langle S_{\text{lipid}} \rangle$ was included because disturbing an ordered bilayer is likely to require more energy that disturbing a disordered one. As a first approximation, $\langle S_{\text{lipid}} \rangle$ was used instead of the lipid disorder around the helix because $\langle S_{\text{lipid}} \rangle$ was more accessible experimentally. Below the gel–fluid transition temperature (T_c), the bilayer may have more than one order parameter [27]. This possibility is not explicitly accounted for in Eq. (2), so the analyses was restricted to cases where the temperature is above T_c, or to comparative studies between two peptides in the same phospholipid at similar temperatures.

Using functions other than the ones in Eq. (2) could be needed for lipid–protein interactions. As mentioned in [24], if no satisfactory fit could be found with γ being a constant, the standard theory is clearly deficient. In this study, we take into account for the other terms inserting a parameter called "the entropic index" mentioned later, by using Tsallis statistics. So there are two parameters, γ and the entropic index, to be fitted to experiment. We must emphasize at this stage that the entropic index parameter controls only the long-range interactions. Some additional comments about the standard theory were: although the theory has been formulated for a single transmembrane helix, it can be used to analyze transmembrane proteins with multiple helices. The average tilt angle for all helices in the same protein has been investigated in [24]. However, the value obtained for γ will then be an average over all different helices in the protein, and may not be meaningfully compared to the value obtained from samples of identical helices. The integration in Eq. (1) accounts for the orientation entropy of the helix and so it could not represent the real system. Because Eq. (2) is chosen somewhat arbitrarily, the significance of γ will only be qualitative.

Now we give the definitions of Tsallis statistics, as well as its axioms.

In 1988, Tsallis proposed a q-dependent entropy functional [28] which generalizes the standard Boltzmann–Gibbs definition, to include non-extensive systems. In this formalism, q is called the entropic index, and it is a measure of the nonextensivity of the system. A few years later, Curado and Tsallis revised the formalism introducing the unnormalized constraint to the internal energy and showed that from this entropy functional a nonextensive thermostatistics could be derived [29]. This generalization included the classical thermostatistics (Boltzmann–Gibbs statistics) which could be recovered when $q \to 1$. After Curado and Tsallis [29], this formalism has been successfully applied to some physical systems, where Boltzmann–Gibbs statistics is known to fail. Some of them could be given such as: self-graviting systems [30,31], turbulence [32–35], anomalous diffusion [36–39], velocities of galaxies [40], solar neutrinos [41], liquid crystals and dimers [42–45] and thermal desorption [46,47].

In spite of these successes, some drawbacks were identified in the early formalism: (i) the density operator was not invariant under a uniform translation of energy spectrum, (ii) energy was not conserved, (iii) the q-expected value of the identity operator was not equal to unity. In a recent study, Tsallis et al. [48] circumvented these difficulties introducing the normalized constraint to the internal energy of the system.

Tsallis entropy is defined to be [28,29,48]

$$S_q = Tr\{\hat{\rho}^q \hat{S}_q\} = \frac{k}{1-q}[Tr\{\hat{\rho}^q\} - 1], \tag{3}$$

where

$$\hat{S}_q = -k \ln_q \hat{\rho} \tag{4}$$

and k is a positive constant. The generalized logarithm is given by [49]

$$\ln_q x = \frac{(x^{1-q} - 1)}{1-q}. \tag{5}$$

The corresponding density operator $\hat{\rho}$ can be determined from the maximization of the S_q imposed to q-normalized constraint [48],

$$U_q = \frac{Tr\{\hat{\rho}^q \hat{H}\}}{Tr\{\hat{\rho}^q\}}, \tag{6}$$

and the normalization of density operator $Tr\{\hat{\rho}\}=1$, resulting in:

$$\hat{\rho}=\frac{1}{Z_q}\left[1-(1-q)\frac{\beta}{Tr\{\hat{\rho}^q\}}(\hat{H}-U_q)\right]^{\frac{1}{1-q}}, \tag{7}$$

where Z_q is the generalized partition function and given by

$$Z_q=Tr\left\{\left[1-(1-q)\frac{\beta}{Tr\{\hat{\rho}^q\}}(\hat{H}-U_q)\right]^{\frac{1}{1-q}}\right\}, \tag{8}$$

where β is the Lagrange parameter associated to internal energy constraint [48,50]. Notice that Eq. (7) is invariant under a uniform translation of the energy spectrum, i.e., the thermodynamic properties do not depend on the choice of the zero of the energy spectrum. In this case, the entropy has a well-defined concavity for any q value being concave for $q>0$ and being convex for $q<0$. Another interesting relation is

$$Tr(\hat{\rho}^q)=Z_q^{1-q},$$

which can be obtained using Eqs. (7) and (8). Furthermore, in Eq. (7), when $(1-(1-q)\beta Z_q^{1-q}(E_n-U_q))\le 0$ ($\{E_n\}$ represents the set of eigenvalues of Hamiltonian $\hat{H}$), we take $\rho(E_n)=0$, which is known as cut-off condition, in order to retain a consistent probabilistic interpretation. In these expressions, the Lagrange multiplier $\beta=1/kT$ is associated with the constraint U_q. The q parameter is a measure of the nonextensivity of the system under consideration, and in the $q\to 1$ limit the standard statistical mechanics is recovered.

If the entropy $\hat{S}_q$ is defined as proposed in Ref. [51]

$$S_q=\frac{Tr\{\hat{\rho}^q\hat{S}\}}{Tr\{\hat{\rho}^q\}}=\frac{k\left[1-\frac{1}{Tr\{\hat{\rho}^q\}}\right]}{1-q}, \tag{9}$$

the density operator $\hat{\rho}$ can be determined in a similar way as above:

$$\hat{\rho}=\frac{1}{Z_q}[1-(1-q)\beta Tr\{\hat{\rho}^q\}(\hat{H}-U_q)]^{\frac{1}{1-q}}, \tag{10}$$

$$Z_q=Tr\left\{[1-(1-q)\beta Tr\{\hat{\rho}^{\,q}\}(\hat{H}-U_q)]^{\frac{1}{1-q}}\right\}. \tag{11}$$

In this case, the entropic index q is allowed to the interval $0\le q\le 1$ and the entropy exhibits concavity. The density operators emerging from Eqs. (8) and (11) can be rewritten as

$$\hat{\rho}=\frac{1}{Z_q'}[1-(1-q)\beta^*\hat{H}]^{\frac{1}{1-q}}, \tag{12}$$

$$Z_q'=Tr\left\{\left[1-(1-q)\beta^*\hat{H}\right]^{\frac{1}{1-q}}\right\}, \tag{13}$$

where

$$\beta^*=\frac{\beta}{\frac{1}{Tr\{\hat{\rho}^q\}}+(1-q)U_q\beta}. \tag{14}$$

Eq. (14) corresponds to the normalized Tsallis entropy. We suggest that the effective temperature is $T=1/k\beta^*$ and thus the density operator becomes independent of the initial entropy functional.

Then any normalized q-expectation value of any observable $\hat{O}$ is calculated by

$$\langle\hat{O}\rangle_q=\frac{Tr\{\hat{\rho}^q\hat{O}\}}{Tr\{\hat{\rho}^q\}} \tag{15}$$

in Tsallis statistics.

2. Results and discussion

We apply the generalized theory to lactose permease, a 12-α-helices protein that catalyses β-galactoside: H^+ symport [11]. Both $\langle\theta\rangle$ and $\langle S_{\text{lipid}}\rangle$ had been determined at 293 K, and various protein concentrations in a phosphatidylglycerol/phosphatidylethanolamine bilayer [11]. Fig. (2) shows the effect of the nonextensivity on the variation of $\langle\theta\rangle_q$ with $\langle S_{\text{lipid}}\rangle$ for various values of q. The average of the tilt angle has been calculated by Eq. (15).

The curve for $q=1$ represents the standard theory in Ref. [24]. Although the $q=1$ curve seems to be appropriate at least the general features of lipid–helix interactions and to be useful for other lipid–helix systems, the agreement between the theoretical and experimental values for $q=1$ deteriorated towards higher values of $\langle S_{\text{lipid}}\rangle$. As mentioned by the author in Ref. [24], this could be due to interactions neglected in Eq. (2). Thus, the cases with $q\neq 1$ could be useful to study the systems like this. However, we must note that the number of experimental data is so few that the results obtained by employing the generalized theory might not be reliable. Nevertheless, we could agree with the author in Ref. [24] that neglecting the other terms in Hamiltonian could be responsible for the disagreement towards higher

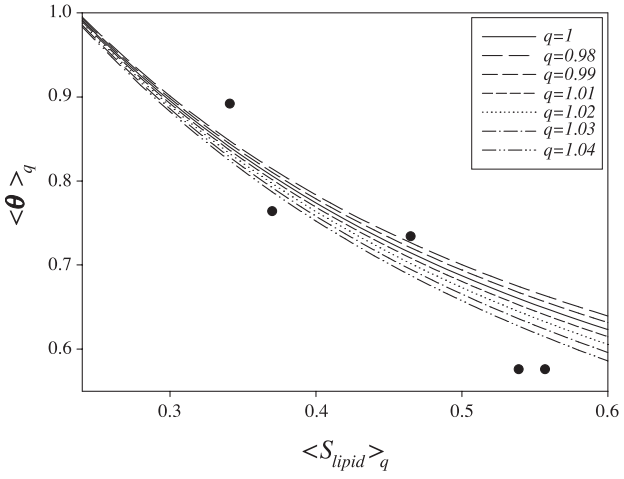

Fig. 2. The dependence of the helix tilt angle $\langle\theta\rangle$ on the average disorder parameter $\langle S_{lipid}\rangle$ of the lipid bilayer for various values of the entropic index q with $\gamma=7.2$. The circles represent the experimental data taken from [11].

values of $\langle S_{\text{lipid}} \rangle$ and the standard theory could be improved employing the nonextensive formalism, namely Tsallis statistics, for various values of q. It is interesting to note that as q increases, the helix tilt angle decreases with $\langle S_{\text{lipid}} \rangle$. Taking q values different from unity means that we are taking into account the long-range interactions. We must also note that so far there have been no evidence whether the systems like the ones considered in Ref. [24] and in this study are nonextensive or not.

The nonextensivity appears in systems where long-range interactions and/or fractality exist, and such features have been invoked in recent models of manganites [52,53], as well as in the interpretation of experimental results. They appear, for instance, in the work of Dagotto et al.'s [54], who emphasize the role of the competition between different phases to the physical properties of these materials. Various authors have considered the formation of microclusters of competing phases, with fractal shapes, randomly distributed in the material [55,56], and the role of long-range interactions to phase segregation [57,58]. In recent studies, Reis et al. [52,53] pursued based on theoretical and experimental results [17,21] that manganites are magnetically nonextensive materials. Thus, we also point out, from another point of view, that future experiments could shed light whether nonextensivity appears in proteins and similar ones.

3. Conclusion

The standard theory presented in Ref. [24] provides a method to compare the rotational barriers for different helices in lipid bilayers. It has been pointed out there that future possible experiments could shed light on lipid–protein interactions. In this study, we give possible contributions of the terms to Hamiltonian given by Eq. (2), using a nonextensive statistics. Also, we would like to open a discussion whether proteins and similar systems exhibit nonextensivity or not. A first attempt has been recently made in Ref. [43] where the effects of the nonextensivity on the dimerization process have been investigated by using Tsallis thermostatistics. Therefore, some extentional studies of similar systems in which one employs Tsallis statistics could be expected.

References

[1] D.R. Fattal, A. Ben-Shaul, A molecular model for lipid–protein interactions in membranes: the role of hydrophobic mismatch, Biophys. J. 65 (1993) 1795–1808.
[2] D.A. Pink, D. Chapman, Protein–lipid interactions in bilayer membranes: a lattice model, Proc. Natl. Acad. Sci. U. S. A. 76 (1979) 1542–1546.
[3] O.G. Mouritsen, M. Bloom, Models of lipid–protein interactions in membranes, Annu. Rev. Biophys. Biomol. Struct. 22 (1993) 145–171.
[4] P.J. Park, W. Sung, Statistical mechanics of membrane protein conformation: a homopolyer model, Phys. Rev. Lett. 80 (1998) 5687–5690.
[5] K. Belochorcová, J.H. Davis, T.B. Woolf, B. Roux, Structure and dynamics of an amphiphilic peptide in a lipid bilayer: a molecular dynamics study, Biophys. J. 73 (1997) 3039–3055.
[6] M. Milik, J. Skolncik, Spontaneous insertion of polypeptide chains in membranes: a Monte Carlo model, Proc. Natl. Acad. Sci. U. S. A. 89 (1992) 9391–9395.
[7] J. Wang, A. Pullman, Do helices in membranes prefer to form bundles or stay dispersed in the lipid phase? Biochim. Biophys. Acta 1070 (1991) 493–496.
[8] A. Baumgärtner, Insertion and hairpin formation of membrane proteins: a Monte Carlo study, Biophys. J. 71 (1996) 1248–1255.
[9] L.R. Forrest, W.F. De Grado, G.R. Dieckmann, M.S.P. Sansom, Two models of the influenza A M2 channel domain: verification by comparison, Fold. Des. 3 (1998) 443–448.
[10] N. Ben-Tal, B. Honig, R.M. Peitzsch, G. Denisov, S. McLaughlin, Binding of small basic peptides to membranes containing acidic lipids: theoretical models and experimental results, Biophys. J. 71 (1996) 561–575.
[11] J. Le Coutre, L.R. Narasimhan, C. Kumar, N. Patel, R. Kaback, The lipid bilayer determines helical tilt angle and function in lactose permease of *Escherichia coli*, Proc. Natl. Acad. Sci. U. S. A. 94 (1997) 10167–10171.
[12] F.A. Kovacs, J.K. Denny, Z. Song, J.R. Quine, T.A. Cross, Helix tilt of the M2 transmembrane peptide from influenza A virus: an intrinsic property, J. Mol. Biol. 295 (2000) 117–125.
[13] S.J. Opella, F.M. Marassi, J.J. Gesell, A.P. Valente, Y. Kim, M. Oblatt-Montal, M. Montall, Structures of the M2 channel-lining segments from nicotinic acetylcholine and NMDA receptors by NMR spectroscopy, Nat. Struct. Biol. 6 (1999) 374–379.
[14] P.J. Booth, Folding α-helical membrane proteins: kinetic studies on bacteriorhodopsin, Fold. Des. 2 (1997) R85–R92.
[15] D. Rice, E. Oldfield, Deuterium nuclear magnetic resonance studies of the interaction between dimyristoylphosphatidylcholine and gramicidin A, Biochemistry 18 (1979) 3272–3279.
[16] F. Jähnig, H. Vogel, L. Best, Unifying description of the effect of membrane proteins on lipid order. Verification for the melittin/dimyristoylphosphatidylcholine system, Biochemistry 21 (1982) 6790–6798.
[17] W. Hoffmann, D. Pink, C. Restall, D. Chapman, Intrinsic molecules in fluid phospholipid bilayers: fluorescence probe studies, Eur. J. Biochem. 114 (1981) 585–589.
[18] P. Meier, J.-H. Sachse, P.J. Brophy, D. Marsh, G. Kothe, Integral membrane proteins significantly decrease the molecular motion in lipid bilayers: a deuteron NMR relaxation study of membranes containing myelin proteolipid apoprotein, Proc. Natl. Acad. Sci. U. S. A. 84 (1987) 3704–3708.
[19] M.R.R. de Planque, D.V. Greathouse, R.E. Koeppe II, H. Schafer, D. Marsh, J.A. Killian, Influence of lipid/protein hydrophobic mismatch on the thicjness of diacylphospatidylcholine bilayers. A ^{2}H NMR and ESR study using designed tranmembrane α-helical peptides and gramicidin A, Biochemistry 37 (1998) 9333–9345.
[20] R.S. Prosser, J.H. Davis, Dynamics of an integral membrane peptide: a deuterium NMR relaxation study of gramicidin, Biophys. J. 66 (1994) 1429–1440.
[21] S.H. White, W.C. Wimley, Membrane protein folding and stability: physical principles, Annu. Rev. Biophys. Biomol. Struct. 28 (1999) 319–365.
[22] G. VonHeijne, Membrane proteins: from sequence to structure, Annu. Rev. Biophys. Biomol. Struct. 23 (1994) 167–192.
[23] C. Glabuitz, G. Gröbner, A. Watts, Structural and orientational information of the membrane embedded M13 coat protein by ^{13}C-MAS NMR spectroscopy, Biochim. Biophys. Acta 1463 (2000) 151–161.
[24] L. Li, Relating helix tilt in a bilayer to lipid disorder: a mean-field theory, Biophys. Chem. 86 (2000) 79–83.
[25] A.Y. Grosberg, A.R. Khokhlov, Statistical Physics of Macromolecules, American Institute of Physics Press, New York, 1994.

[26] J. Seeling, Deuterium magnetic resonance: theory and application to lipid membranes, Q. Rev. Biophys. 10 (1977) 353–418.
[27] M. Rance, K.R. Jeffrey, A.P. Tulloch, K.W. Butler, I.C.P. Smith, Orientational order of unsaturated lipids in the membranes of *Acholeplasma laidlawii* as observed bu ^{2}H-NMR, Biochim. Biophys. Acta 600 (1980) 245–262.
[28] C. Tsallis, Possible generalization of Boltzmann–Gibbs Statistics, J. Stat. Phys. 52 (1988) 479.
[29] E. Curado, C. Tsallis, Generalized statistical-mechanics—connection with thermodynamics, J. Phys. A: Math. Gen. 24 (1991) L69.
[30] A.R. Plastino, A. Plastino, Stellar polytropes and Tsallis entropy, Phys. Lett., A 174 (1993) 384.
[31] J.J. Aly, in: F. Combes, E. Athanassoula (Eds.), Proceedings of N-Body Problems and Gravitational Dynamics, Publications de L'Observatoire de Paris, Paris, 1993, p. 19.
[32] T. Arimitsu, N. Arimitsu, Analysis of fully developed turbulence in terms of Tsallis statistics, Phys. Rev., E 61 (2000) 3237.
[33] T. Arimitsu, N. Arimitsu, Tsallis statistics and fully developed turbulence, J. Phys. A 33 (2000) L235.
[34] C. Beck, Physica, A 277 (2000) 115.
[35] C. Beck, G. Lewis, H. Swinney, Phys. Rev., E 63 (2001) 035303.
[36] A. Plastino, A. Plastino, Non-extensive statistical mechanics and generalized Fokker–Planck equation, Physica, A 222 (1995) 347.
[37] C. Tsallis, D. Bukman, Anomalous diffusion in the presence of external forces: exact time-dependent solutions and their thermostatistical basis, Phys. Rev., E 54 (1996) R2197.
[38] M. Bologna, C. Tsallis, P. Grigolini, Anomalous diffusion associated with nonlinear fractional derivative Fokker–Planck-like equation: exact time-dependent solutions, Phys. Rev., E 62 (2000) 2213.
[39] L. Malacarne, L. Pedron, R. Mendes, E. Lenzi, Phys. Rev., E 63 (2001) 30101R.
[40] A. Lavagno, G. Kaniadakis, M. Rego-Monteiro, P. Quarati, C. Tsallis, Non-extensive thermostatistical approach of the peculiar velocity function of galaxy clusters Astro, Lett. Comm. 35/6 (1998) 449.
[41] A. Lavagno, P. Quarati, Nucl. Phys., B 87 (2000) 209.
[42] O. Kayacan, F. Büyükkilic, D. Demirhan, Generalization of the Maier–Saupe theory of the nematics within Tsallis thermostatistics, Physica, A 301 (2001) 255.
[43] O. Kayacan, The effects of the nonextensivity on the dimerization process and nematic ordering, Physica, A 325 (2003) 205.
[44] O. Kayacan, Mean-field theory of anisotropic potential of rank $L=4$ and nonextensive formalism, Chem. Phys. 297 (2004) 1.
[45] O. Kayacan, Orientational ordering in the nematic phase of a thermotropic liquid crystal: a nonextensive approach, Physica, A 337 (2004) 123.
[46] O. Kayacan, N. Can, Nonextensive thermostatistical approach to the thermoluminescence decay, Physica, A 331 (2004) 538.
[47] O. Kayacan, N. Can, A generalized method for handling first- and second-order thermal desorption and thermoluminescence, Chem. Phys. 298 (2004) 273.
[48] C. Tsallis, R. Mendes, A. Plastino, The role of constraints within generalized nonextensive statistics, Physica, A 261 (1998) 534.
[49] E. Borges, On a q-generalization of circular and hyperbolic functions, J. Phys. A: Math. Gen. 31 (1998) 5281.
[50] C. Tsallis, Nonextensive Statistical Mechanics and Its Applications, Springer-Verlag, Heidelberg, 2001.
[51] A. Rajagopal, S. Abe, Implications of form invariance to the structure of nonextensive entropies, Phys. Rev. Lett. 83 (1999) 1711.
[52] M.S. Reis, J.P. Araũjo, V.S. Amaral, E.K. Lenzi, I.S. Oliveira, Magnetic behavior of a nonextensive S-spin system: possible connections to manganites, Phys. Rev., B 66 (2002) 134417.
[53] M.S. Reis, J.C.C. Freitas, M.T.D. Orlando, E.K. Lenzi, I.S. Oliveira, Evidences for Tsallis non-extensivity on CMR manganites, Europhys. Lett. 58 (2002) 42.
[54] E. Dagotto, T. Hotta, A. Moreo, Colossal magnetoresistant materials: the key role of phase separation, Phys. Rep. 344 (2001) 1.
[55] M. Mayr, A. Moreo, J.A. Verges, J. Arispe, A. Feiguin, E. Dagotto, Resistivity of mixed-phase manganites, Phys. Rev. Lett. 86 (2001) 135.
[56] A.L. Malvezzi, S. Yunoki, E. Dagotto, Influence of nearest-neighbor Coulomb interactions on the phase diagram of the ferromagnetic Kondo model, Phys. Rev., B 59 (1999) 7033.
[57] A. Moreo, S. Yunoki, E. Dagotto, Solid state physics—phase separation scenario for manganese oxides and related materials, Science 283 (1999) 2034.
[58] J. Lorenzana, C. Castellani, C.D. Castro, Phase separation frustrated by the long-range Coulomb interaction: I. Theory, Phys. Rev., B 64 (2001) 235127.